Model of Balanced Transportation Roads for Reduced Traffic: *A Quantitative Model Approach to Address Traffic Congestion*

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Abstract

Traffic congestion, due to increase in demand for highway use, has exponentially increased in the last few years. With automobiles increasing day by day, there are more cars on road year by year. The complex and dynamic behavior of the commuters based on their schedules and routes influencing traffic congestion, is sometimes addressed by widening of roadways and increase in construction and is found to be non-feasible.

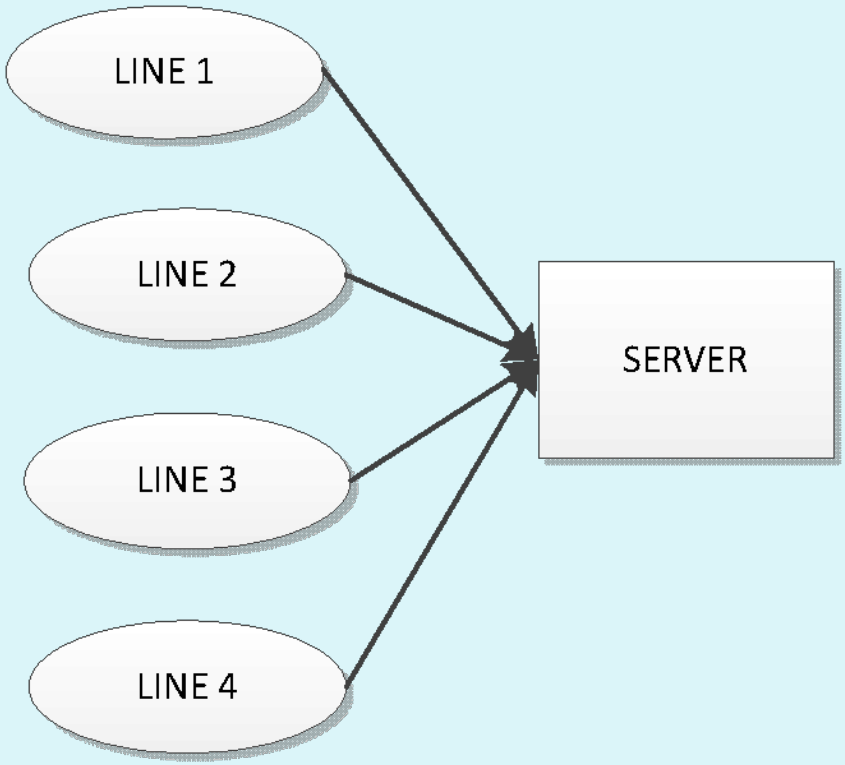
- **Quantitative approach** by contemplating the problem of traffic congestion as a complex network without a proper balance in rate of input and outputs. This model also addresses the problem of highway alignment optimization. The developed algorithm is based on queuing theory and ant colony optimization techniques and aims to search for an optimal path in any given complex situation along with inherent capability of reducing the time factor involved based on the formulated inputs which correspond to rate of arrivals and highway service. The rate of input based on poison distribution and the corresponding service rate based on exponential distribution are the two most important criteria of this algorithm.

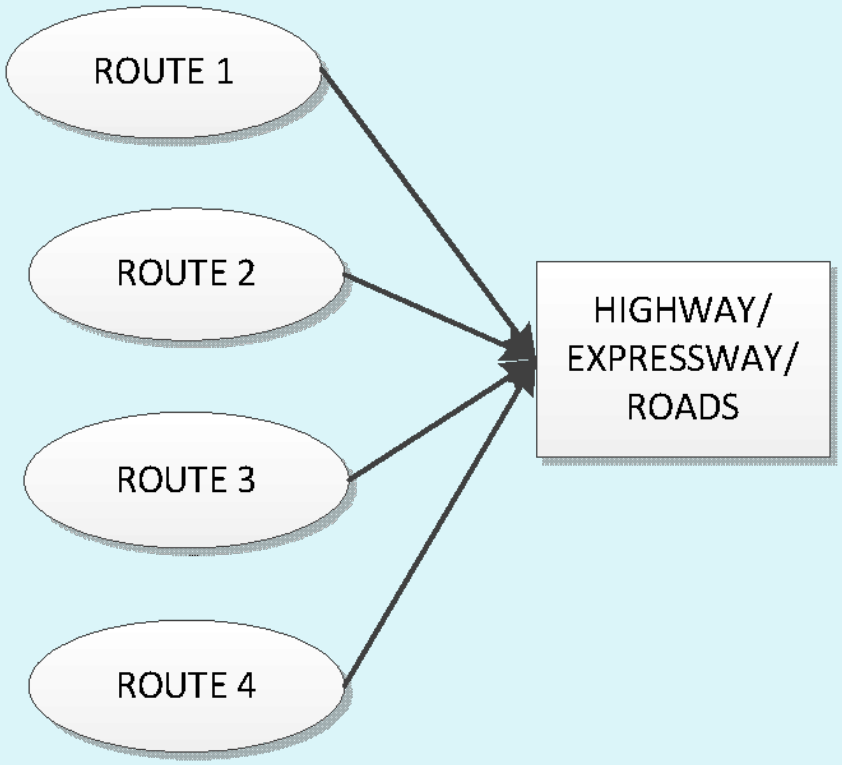
Algorithm basis

- t_i = Passing time
- d_i = Distribution in i area

$$\frac{d_i \frac{1}{t_i}}{\sum_{i=1}^{n} \frac{1}{t_i} d_i} \qquad (1)$$

Queuing theory in Networks and in Transportation





Primary Quantitative Model

- **Assumptions**
- a) Rate of entry based on Equivalent Poisson distribution
- b) Rate of exit based on Equivalent Exponential distribution
- c) System is based on assumption of one server
- d) System has no limit on number of entry lines
- e) Server has a continuous service
- **Index:** i: Number of entry lines
- **Parameter:** n: total no: entry lines
- **Variable:** μ: Standard service rate

(2) Summation of input paths

$$\frac{d_1 \frac{1}{c_1}}{\sum_{i=1}^{1} d_1 \frac{1}{c_1}} + \frac{d_2 \frac{1}{c_2}}{\sum_{i=1}^{2} d_2 \frac{1}{c_2}} + \frac{d_3 \frac{1}{c_3}}{\sum_{i=1}^{3} d_3 \frac{1}{c_3}} + \frac{d_4 \frac{1}{c_4}}{\sum_{i=1}^{4} d_4 \frac{1}{c_4}} + \cdots$$

(3) Equivalent Poisson distribution with d_i is represented as

$$d_i = \frac{\lambda_i^{x} e^{-\lambda_i}}{x!}$$

(4) (5) Considering the vehicles in the system to be in a queue and substituting d_i

$$\sum_{i=1}^{n} d_i \frac{1}{c_i} = \sum_{i=1}^{n} \frac{1}{C_i} \frac{\lambda_i{}^{x} e^{-\lambda i}}{\mathrm{x}!}$$

$$\sum_{i=1}^{n} \frac{\frac{d_i}{c_i}}{\sum_{i=1}^{n} \frac{d_i}{c_i}} = \sum_{i=1}^{n} \frac{\frac{\lambda_i{}^{x} e^{-\lambda i}}{\mathrm{x}!} \frac{1}{c_i}}{\sum_{i=1}^{n} \frac{\frac{\lambda_i{}^{x} e^{-\lambda i}}{\mathrm{x}!}}{c_i}}$$

Second Quantitative model: (6)

Equivalent Exponential distribution with d_i

$$d_i = \frac{1}{\beta} e^{-\frac{x}{\beta}}$$

(7) Considering that the service of the system is continuous …

$$d_i = \int \frac{1}{\beta e^{\frac{x}{\beta}}}$$

(8) Substitution of in equation 2 …

$$\sum_{i=1}^{n} \frac{\frac{d_i}{c_j}}{\sum_{i=1}^{n} \frac{d_i}{c_j}} = \frac{\int \frac{1}{\beta} e^{\frac{-x}{\beta}} \frac{1}{c_j}}{\sum_{i=1}^{n} \frac{1}{c_i} \int \frac{e^{\frac{-x}{\beta}}}{\beta}}$$

(9) Considering the maximum limit of vehicles allowed …

$$\sum_{i=1}^{n} \frac{\frac{d_i}{c_j}}{\sum_{i=1}^{n} \frac{d_i}{c_j}} = \frac{\frac{\int_0^a \frac{1}{\beta e^{\frac{x}{\beta}}} dx}{c_j}}{\sum_{i=1}^{n} \frac{\int_0^a \frac{1}{\beta e^{\frac{x}{\beta}}} dx}{c_i}}$$

(10) Equating (9) and (5) to obtain the power of the server or highway we have …

$$\sum_{i-1}^{n} \frac{\frac{\lambda i^{x} e^{-\lambda i}}{x_i} \frac{1}{c_i}}{\sum_{i=1}^{n} \frac{\lambda i^{x} e^{-\lambda i}}{x_i} \frac{1}{c_i}} = \frac{\frac{\int_0^{a} \frac{1}{\beta e^{\frac{x}{\beta}}} dx}{c_j}}{\sum_{i=1}^{n} \frac{\int_0^{a} \frac{1}{\beta e^{\frac{x}{\beta}}} dx}{c_i}}$$

For c_i being the average number of customers waiting the system and c_i being the average waiting time in the system (11) (12)

$$c_i = \frac{\lambda}{\mu(\mu - \lambda_i)}$$

$$c_j = \frac{1}{(\mu - \lambda_j)}$$

(13) Using the average waiting times …

$$\sum_{i-1}^{n} \frac{\dfrac{\lambda i^{x} e^{-\lambda i}}{x_i} \dfrac{1}{\dfrac{\lambda_i}{\mu(\mu-\lambda_i)}}}{\sum_{i=1}^{n} \dfrac{\lambda i^{x} e^{-\lambda i}}{x_i} \dfrac{1}{\dfrac{\lambda_i}{\mu(\mu-\lambda_i)}}} = \frac{\dfrac{\int_0^{a} \dfrac{1}{\beta e^{\frac{x}{\beta}}} dx}{\dfrac{1}{(\mu-\lambda_j)}}}{\sum_{i=1}^{n} \dfrac{\int_0^{a} \dfrac{1}{\beta e^{\frac{x}{\beta}}} dx}{\dfrac{1}{(\mu-\lambda_i)}}}$$

(14)

$$\sum_{i=1}^{n} \frac{\frac{(\lambda i^{x} e^{-\lambda i})}{x_i} \frac{\mu(\mu-\lambda_i)}{\lambda_i}}{\sum_{i=1}^{n} \frac{\lambda i^{x} e^{-\lambda i}}{x_i} \frac{\mu(\mu-\lambda_i)}{\lambda_i}} = \frac{(\mu-\lambda_j)\int_0^{a} \frac{1}{\beta e^{\frac{x}{\beta}}} dx}{\sum_{i=1}^{n} (\mu-\lambda_i)\int_0^{a} \frac{1}{\beta e^{\frac{x}{\beta}}} dx}$$

(15) Assuming =1

$$\sum_{i=1}^{n} \frac{\frac{(\lambda_i{}^{x} e^{-\lambda_i})}{x_i} \frac{\mu(\mu-\lambda_i)}{\lambda_i}}{\sum_{i=1}^{n} \frac{\lambda_i{}^{x} e^{-\lambda_i}}{x_i} \frac{\mu(\mu-\lambda_i)}{\lambda_i}} = \frac{(\mu-\lambda_j)\int_0^{\alpha} \frac{1}{e^{x}\,dx}}{\sum_{i=1}^{n} (\mu-\lambda_i)\int_0^{\alpha} \frac{1}{e^{x}\,dx}}$$

(16)

$$\sum_{i=1}^{n} \frac{\frac{(\lambda_i{}^{x} e^{-\lambda_i})}{x_i} \frac{\mu(\mu-\lambda_i)}{\lambda_i}}{\sum_{i=1}^{n} \frac{\lambda_i{}^{x} e^{-\lambda_i}}{x_i} \frac{\mu(\mu-\lambda_i)}{\lambda_i}} = \frac{(\mu-\lambda_j)(1-e^{-\alpha})}{\sum_{i=1}^{n} (\mu-\lambda_i)(1-e^{-\alpha})}$$

Based on the above equation 16,
given respective input parameters,
one can obtain the value of μ.
This value can be useful to schedule the
rate of service on a given highway.

Conclusion

- The model developed in this paper is a feasible approach to solve traffic congestion as it can be programmable to adjust the rate of entry and rate of exit to a highway. This model can also be used for decision making based on the rate of distribution corresponding to any considered service area. This decision making model is based on a Poisson distribution for the rate of entry and on an exponential distribution for the rate of exit, both following the ant colony optimization algorithm.

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Questions?

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