A Model for Evaluating Effectiveness of Modular Systems

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Background

- In 1963, the Weapon System Effectiveness Industry Advisory Committee (WSEIAC) defined **system effectiveness** as:

  "…a measure of the extent to which a system may be expected to achieve a set of specific mission requirements and is a function of *availability, dependability and capability.*"

- The system effectiveness model is based on assessing availability (A), dependability (D), and capability (C) for all possible system states.

$$\mathrm{SE} = ADC$$

- This produces an availability row vector, dependability matrix, and capability matrix, which are then multiplied together to produce a vector that describes the overall effectiveness of the system.

System States

- Consider a modular power system comprised of three independently functioning component power sources:
  - Generator, solar panel, and battery
- Each component can either be in an "1" or "0" state:
  - "1" : component is operable
  - "0" : component is non-operable

State	Generator	Solar Panel	Battery
1	1	1	1
2	0	1	1
3	1	0	1
4	1	1	0
5	0	0	1
6	1	0	0
7	0	1	0
8	0	0	0

Availability

- Availability is expressed as a row vector:

$$A = (a_1, a_2, a_3, \dots, a_i, \dots, a_n)$$

  where, a_i is the probability that the system will be in State i at a randomly chosen mission *start* time.

- a_i is a product of operational availabilities for each of the three system components for State i

Example:

$$a_2 = \left(\frac{DT}{UT + DT}\right)_{Gen} * \left(\frac{UT}{UT + DT}\right)_{Sol} * \left(\frac{UT}{UT + DT}\right)_{Bat}$$

where, $DT = downtime;\ UT = uptime;$

System Types

- Three types of systems - formulation of the dependability matrix is different for each:
  1. Type 1 – example: guided missile
     - Repair not possible during mission.
     - Mission success determined by system state at end of mission (i.e. did the missile hit the target?)
  2. Type 2 – example: supply delivery
     - Repair is possible during mission.
     - Mission success determined by system state at end of mission (i.e. did the vehicle arrive at final destination?)
  3. **Type 3 – example: power system**
     - Repair is possible during mission.
     - Mission success determined by fraction of mission time which system is operational (i.e. did the generator supply continuous power?)

Component Dependability

- Each component can only be operable or failed at the start of a mission, thus yielding a 2x2 matrix for each component of the following form:

$$D_c = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

Where,

- $\boldsymbol{d_{11}} =$ expected fraction of mission time during which the component is *operable*, given that it was *operable* at the start.
- $\boldsymbol{d_{12}} =$ expected fraction of mission time during which the component is *failed*, given that it was *operable* at the start.
- $\boldsymbol{d_{21}} =$ expected fraction of mission time during which the component is *operable*, given that it was *failed* at the start.
- $\boldsymbol{d_{22}} =$ expected fraction of mission time during which the component is *failed*, given that it was *failed* at the start.

Component Dependability

- Assume:
  - Probability of failure in time Δt: $\lambda \Delta t$
  - Probability of repair in time Δt: $\mu \Delta t$
- Define:

$P_1(t) =$ Prob. that a component is operable at time t

$P_2(t) =$ Prob. that a component is failed at time t

- Write two equations:

(1) $$P_1(t + \Delta t) = P_1(t)(1 - \lambda \Delta t) + P_2(t)\mu \Delta t$$

(2) $$P_1(t) + P_2(t) = 1$$

Component Dependability

Component dependability matrix formulas:

$$d_{11} = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2 T}\left[1 - e^{-(\lambda+\mu)T}\right]$$

$$d_{12} = 1 - d_{11} = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{(\lambda + \mu)^2 T}\left[1 - e^{-(\lambda+\mu)T}\right]$$

$$d_{21} = \frac{\mu}{\lambda + \mu} * \left[1 - \frac{1}{(\lambda + \mu)T}\left[1 - e^{-(\lambda+\mu)T}\right]\right]$$

$$d_{22} = 1 - d_{21} = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{(\lambda + \mu)^2 T}\left[1 - e^{-(\lambda+\mu)T}\right]$$

Where:

$$T = Mission\ Duration$$

System Dependability

- Using logic similar to that for component dependability, system dependability is expressed as an 8x8 matrix of the following form:

$$D = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{18} \\ d_{21} & d_{22} & \dots & d_{28} \\ \vdots & \vdots & \ddots & \vdots \\ d_{81} & d_{82} & \dots & d_{88} \end{bmatrix}$$

  where, d_{ij} is the expected fraction of mission time during which the system will be in State j if it were in State i at the beginning of the mission.

- d_{ij} is a product of elements of the component dependability matrices. Example:

  $d_{12} =$ fraction of mission time the system spends in State 2 given that it began the mission in State 1

  $$d_{12} = d_{12(Gen)} * d_{11(Sol)} * d_{11(Batt)}$$

Capability

- Define element(s) of mission success.
  - Example: fuel consumption below a given threshold
- Capability is then expressed as a column matrix:

$$C = \begin{bmatrix} c_{11} \\ c_{21} \\ \vdots \\ c_{n1} \end{bmatrix}$$

where, c_{i1} is the performance of the system in State i

- Each additional element of mission success is handled by adding columns to the capability matrix.

System Effectiveness

$$SE = ADC = \begin{bmatrix} a_1 & a_2 & \dots & a_8 \end{bmatrix} \begin{bmatrix} d_{11} & d_{12} & \dots & d_{18} \\ d_{21} & d_{22} & \dots & d_{28} \\ \vdots & \vdots & \ddots & \vdots \\ d_{81} & d_{82} & \dots & d_{88} \end{bmatrix} \begin{bmatrix} c_{11} \\ c_{21} \\ \vdots \\ c_{81} \end{bmatrix} = [SE]$$

- Model produces a n-element row vector that describes the overall performance of the system in regards to each of the n elements of mission success:
- Model results can be compared directly to threshold values to assess system effectiveness.

Model Assumptions

- Each system component fails independent of the other components.
- Repair time and time between component failures are exponentially distributed with constant rates.
- Failures are immediately evident and repairs are initiated right away.

Model Example

Note: Data is notional

Availability	Generator	Solar	Battery
Up (hrs)	500	500	500
Down (hrs)	20	10	2

Availability Matrix

A1	A2	A3	A4	A5	A6	A7	A8
0.9389	0.0376	0.0188	0.0038	0.0008	0.0001	0.0002	0.0000

Depend.	Generator	Solar	Battery	Mission Time
λ	0.004	0.01	0.002	48
μ	0.5	0.5	0.05	

Dependability Matrix

	D1	D2	D3	D4	D5	D6	D7	D8
D1	0.950	0.007	0.018	0.024	0.000	0.000	0.000	0.000
D2	0.910	0.047	0.017	0.023	0.001	0.000	0.001	0.000
D3	0.910	0.007	0.058	0.023	0.000	0.001	0.000	0.000
D4	0.592	0.005	0.011	0.382	0.000	0.007	0.003	0.000
D5	0.873	0.045	0.055	0.022	0.003	0.001	0.001	0.000
D6	0.567	0.004	0.036	0.366	0.000	0.023	0.003	0.000
D7	0.567	0.029	0.011	0.366	0.001	0.007	0.019	0.000
D8	0.544	0.028	0.034	0.351	0.002	0.022	0.018	0.001

Model Example

Note: Data is notional

Capability Matrix

	Fuel Consumption (gal/day)
C1	2.7
C2	0.0
C3	4.5
C4	6.1
C5	0.0
C6	8.2
C7	0.0
C8	0.0

$$SE = ADC$$

↓

	Fuel Consump.
System Effectiveness	2.797

↓

Compare results directly to thresholds.

Challenges to Implementation

- Handling nighttime inoperability of the solar panel. (Is it downtime or a failure?)

- Fuel consumption rate varies as a function of power demand and operating time.

- Ability of operational testing to satisfy data requirements of the capability matrix. (Not all system states may be experienced).

References

- Weapon System Effectiveness Industry Advisory Committee (WSEIAC). “Final Report of Task Group II: Prediction – Measurement (Concepts, Task Analysis, Principles of Model Construction”, Doc #AF-TR-65-2, Pages 22-34. 1965.

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