How to Use Firepower of Blue and Red Weapons in an Operational Test?

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Introduction

Battle

Blue Force

#Blue Losses

(killed by Enemy)

Red Force

#Red Losses

(killed by Friendly)

Loss Exchange Ratio (LER) = (number of Red losses / number of Blue losses)

Who is winning the battle?

Traditional Analysis

Friendly force is winning the battle if LER is greater than one.

Enemy force is winning the battle if LER is less than one.

Drawback of Traditional Analysis

Every weapon destroyed is counted equally, regardless of its lethality

EXAMPLE

Friendly losses: 10 Armored Personnel Carrier (APC)
Enemy losses: 5 tanks

Who is winning the battle?

By traditional analysis, enemy is winning since enemy got fewer losses than the friendly.
LER (= 5/10 = 0.5) is less than one

Proposed Analysis

Previous Example: The firepower of 10 APC is less than the firepower of 5 tanks

Who is winning the battle?

In the proposed analysis, friendly is winning since the 5 tanks lost by the enemy got higher firepower than the 10 APC lost by the friendly.

Firepower

Webster's Dictionary:

The Capacity to deliver effective fire on a target

Firepower Cont.

Proposed definition:

Firepower of a Blue or Red weapon is determined by both the *quantity* and *lethality* of opposing weapons it destroyed

Mathematical Derivation of Firepower

Blue Forces: $B_1, B_2, B_3, \ldots, B_b$

Red Forces: $R_1, R_2, R_3, \ldots, R_r$

where b is the number of *types of Blue weapons* and r is the number of *types of Red weapons*.

For example, aircraft (equipped with air-to-ground missiles), missiles (ground-to-ground and ground-to-air), tanks, artillery, and APCs are types of weapons in the Blue/ Red forces.

Mathematical Derivation Cont.

Given: Matrices of total probabilities of kills in an Operational Test (OT), Blue against Red and Red against Blue. In an OT the probabilities of kill are obtained by simulation.

Task: Find the firepower of the weapons in the Blue and Red forces.

Mathematical Derivation Cont.

Blue Kill Matrix: {BK(i,j)}

BK(i,j) = sum of probabilities of kills by the Blue weapon B_i, against the Red weapon R_j (partial kills allowed)

Red Kill Matrix: {RK(i,j)}

RK(i,j) = sum of probabilities of kills by the Red weapon R_i, against the Blue weapon B_j

EXAMPLE OF BLUE KILL MATRIX

Kill Matrix of Blue against Red

BK(i,j)

	Red Missile	Red Tank	Red Artillery	Total
Blue Aircraft	1.5	10.5	5.5	17.5
Blue Tank	0.5	5.5	1.5	7.5
Blue Artillery	0.1	5.0	2.0	7.1
Blue APC	0.0	1.0	0.0	1.0
$BK(R_j)$	2.1	22.0	9.0	33.1

EXAMPLE OF RED KILL MATRIX

Kill Matrix of Red against Blue RK(i,j)

	Blue Aircraft	Blue Tank	Blue Artillery	Blue APC	Blue Total
Red Missile	2.5	2.0	1.0	10.0	15.5
Red Tank	3.5	3.0	0.5	3.5	10.5
Red Artillery	1.0	1.0	3.0	2.1	7.1
$RK(B_j)$	7.0	6.0	4.5	15.6	33.1

Mathematical Derivation Cont.

BF(i) = Firepower of the Blue weapon B_i

= Sum of products of the firepower of the Red weapons killed by B_i and the corresponding total probabilities of kill. TBF is a normalizing factor defined in equation (5).

$$= \sum_{j=1}^{j=r} BK(i,j)RF(j) / TBF \qquad (1)$$

Mathematical Derivation Cont.

RF(i) = Firepower of the Red weapon R_i

= Sum of products of the firepower of the Blue weapons killed by R_i and the corresponding total probabilities of kill. TRF is a normalizing factor defined in equation (6).

$$= \sum_{j=1}^{j=b} RK(i,j)BF(j) / TRF \qquad (2)$$

Solution of Equations (1) and (2)

$BK(R_j)$ = Sum of the probabilities of Blue kills against the Red weapon R_j.

$$= \sum_{i=1}^{i=b} BK(i,j) \qquad (3)$$

$RK(B_j)$ = Sum of the probabilities of Red kills against the Blue weapon B_j.

$$= \sum_{i=1}^{i=r} RK(i,j) \qquad (4)$$

Solution of Equations (1) and (2) Cont.

TBF = Total Blue firepower

= Sum of products of the firepower of the Red weapons and the corresponding Blue kill probabilities aggregated against the Red weapons.

$$= \sum_{j=1}^{j=r} BK(R_j)RF(j) \qquad (5)$$

Solution of Equations (1) and (2) Cont.

TRF = Total Red firepower

= Sum of products of the firepower of the Blue weapons and the corresponding Red kill probabilities aggregated against the Red weapons.

$$= \sum_{j=1}^{j=b} RK(B_j)BF(j) \qquad (6)$$

Solution of the equations (1) and (2) Cont.

Task: Solve the linear equations (1) and (2) for the unknowns BF(i) and RF(i).

Method: Iterative Process by setting initial values of BF(i) to 1/b.

TBF in equation (1) and TRF in equation (2) act as normalizing factors, which guaranty the convergence of the iterative process. An Excel program is used to ease computation.

Final Solution

After getting the normalized solution, multiply BF(i) by TBF and RF(i) by TRF to get the un-normalized solutions.

EXAMPLE Cont.

Kill Matrix of Blue against Red

BK(i,j)

	Red Missile	Red Tank	Red Artillery	Total
Blue Aircraft	1.5	10.5	5.5	17.5
Blue Tank	0.5	5.5	1.5	7.5
Blue Artillery	0.1	5.0	2.0	7.1
Blue APC	0.0	1.0	0.0	1.0
$BK(R_j)$	2.1	22.0	9.0	33.1

EXAMPLE Cont.

Kill Matrix of Red against Blue RK(i,j)

	Blue Aircraft	Blue Tank	Blue Artillery	Blue APC	Total
Red Missile	2.5	2.0	1.0	10.0	15.5
Red Tank	3.5	3.0	0.5	3.5	10.5
Red Artillery	1.0	1.0	3.0	2.1	7.1
$RK(B_j)$	7.0	6.0	4.5	15.6	33.1

EXAMPLE Cont.

Final Solution

Firepower of Blue Force

	Normalized Solution	Un-normalized Solution
BF(1):Blue Aircraft	0.51	6.11
BF(2):Blue Tank	0.24	2.88
BF(3):Blue Artillery	0.22	2.64
BF(4):Blue APC	0.03	0.36
Total	1.00	11.99 = TBF

EXAMPLE Cont.

Final Solution Cont.
Firepower of Red Force

	Normalized Solution	Un-normalized Solution
RF(1): Red Missile	0.35	2.29
RF(2): Red Tank	0.42	2.74
RF(3): Red Artillery	0.23	1.50
Total	1.00	6.53 = TRF

EXAMPLE Cont.

In the example, both the kill matrices have total probability of kills 33.1. However, the total firepower of Blue force (TBF=11.99) is almost twice the total firepower of Red force (TRF=6.53). This is so because close to 50% of the Red kills were against the Blue APC and the APC was the least effective Blue weapon against the Red force.

Application of This Analysis

- **Rank the effectiveness of Blue and Red weapons among themselves (see next slide).**
- **Assess the total performance of the Blue force (TBF) against the Red force (TRF).**
- **Compare the performance of a new weapon against a baseline in an OT, when both of them are tested against the *same Red force*.**

Ranking Blue Weapons

Firepower of Blue Force

	Normalized Solution	Un-normalized Solution
BF(1):Blue Aircraft	0.51	6.11
BF(2):Blue Tank	0.24	2.88
BF(3):Blue Artillery	0.22	2.64
BF(4):Blue APC	0.03	0.36
Total	1.00	11.99 = TBF

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