Comments on RCC-118-02, vol-2, App C - Solar Calibration

10 May 2017

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Overview

- **Introduction**
- **The Range Commander's Council (RCC) Interpolation Formula for Solar Calibration**
- **Solar Surface Radio Frequency (RF) Model**
- **Comparison of Solar Surface RF vs. National Oceanic and Atmospheric Administration (NOAA) Solar Radio Data**
- **Comparison of Interpolation Schemes**
- **Discussion and Recommendations**
- **Derivation of RCC Interpolation Formula**

Introduction

- **The purpose of this document is to report on the findings resulting from a review of certain Edwards Air Force Base (EAFB) gain/temperature (G/T) measurement procedures for telemetry antenna calibration.**

- **Of particular interest for this report is the solar flux density *interpolation* formula, which is widely used at this and many other ranges..**

- **The research task reported herein was in support of telemetry personnel performing refurbishment, sustainment, and calibration of the various Ridley Mission Control Range dish antennas throughout the 412th Test Wing (TW) under the auspices of the Telemetry Systems Integration and Support (TSIS) program.**

Background

- **During an Air Force Test Center / 412th TW telemetry (TM) systems upgrade program at EAFB, the implementation revealed some issues with the solar gain/temperature calibrations for the large parabolic reflector antennas.**

- **Real solar flux data are commonly used for calibration by TM personnel, via access to daily updates of NOAA web site reports.**

- **The problem centered around a calibration step requiring the use of an accepted interpolation formula provided by a RCC TM manual titled *Test Methods for Telemetry RF Systems – App C-Solar Calibration*.**
- **An MS Word report document, with study details, is available from the author on request.**

Solar Cal Interpolation Formula

The RCC-118-02 solar power flux density interpolation formula is

$$S = \left[\frac{S_{1415}}{S_{2695}}\right]^{\Gamma} \bullet S_{2695}$$

where

$$\Gamma = \frac{\log\left[\frac{f_1}{2695}\right]}{\log\left[\frac{1415}{2695}\right]}$$

for which

S = **(S_f) measured power flux density ($Wm^{-2}Hz^{-1}$) at the test frequency, $f_1 = f$;**

S_{2695} = **(S_S) Sagamore Hill power flux density ($Wm^{-2}Hz^{-1} \times 10^{-22}$) at f_S = 2695 MHz;**

S_{1415} = **(S_L) Sagamore Hill power flux density ($Wm^{-2}Hz^{-1} \times 10^{-22}$) at f_L = 1415 MHz.**

Solar Surface RF Thermal Model

- **Solar Optical Radiation Models**
  - **In optical astronomy, the solar surface is modeled as an ideal thermal radiator at 5900K and works quite well.**
  - **Planck radiation law for all frequencies**
    - **L_f, the spectral radiance ($Wm^{-2}sr^{-1}Hz^{-1}$), at the surface of the sun is**

$$L_f = \frac{2hf^3}{c^2} \frac{1}{e^{hf/k_B T_{sun}} - 1}$$

    - **The spectral irradiance, E_f ($Wm^{-2}Hz^{-1}$) at the earth is given by**

$$E_f = \frac{\pi r^2 L_f}{R^2}$$

    - **Astronomers have verified that this formulation works very well across the visible and infrared portion of the electromagnetic solar radiation.**
  - **It is natural to assume the optical model applies to RF frequencies also. If we extend the 5900K model to RF frequencies**

$$S_{RF} = \frac{\pi r^2}{R^2} \frac{2k_B T_{sun}}{c^2} f^2$$

    - **with r = the solar radius (km), R = sun to earth distance (km), f = frequency (Hz), T_{sun} = solar surface temperature (K), c = speed of light (ms^{-1}), and k_B = Boltzmann's constant (JK^{-1}).**
  - **The frequency squared dependence of the formula is the characteristic most germane to the arguments to be made herein.**

NOAA Space Weather Operations Web Site Data

http://legacy-www.swpc.noaa.gov/ftpdir/lists/radio/rad.txt

```
:Product: Solar Radio Data                rad.txt
:Issued: 2232 UTC 06 Mar 2013
#
# Prepared by the U.S. Dept. of Commerce, NOAA, Space Weather Prediction Center
# Please send comments and suggestions to SWPC.Webmaster@noaa.gov
#  Units: 10^-22 W/m^2/Hz
#  Missing Data:  -1
#
#    Daily local noon solar radio flux values - Updated once an hour
#
Freq  Learmonth  San Vito  Sag Hill  Penticton  Penticton  Palehua  Penticton
MHZ   0500 UTC  1200 UTC  1700 UTC   1700 UTC   2000 UTC  2300 UTC  2300 UTC


2013 Mar 5
245       24        20        19         -1         -1        22        -1
410       39        47        40         -1         -1        -1        -1
610       65        -1        62         -1         -1        58        -1
1415      100       103        97         -1         -1        98        -1
2695      110       118       112         -1         -1       109        -1
2800       -1        -1        -1        118        118        -1       117
4995      149       154       146         -1         -1       166        -1
8800      280       268       285         -1         -1       275        -1
15400       -1       537       587         -1         -1       579        -1
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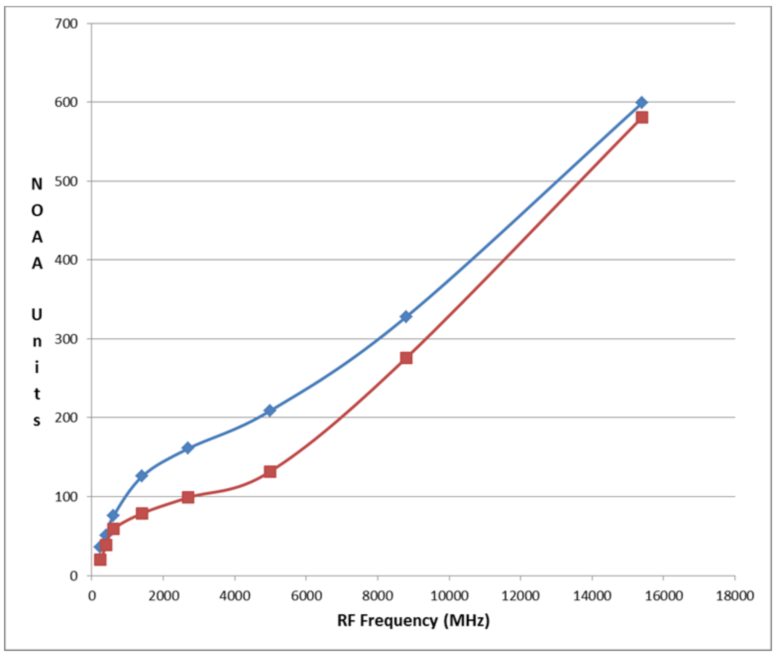
The Real Frequency Behavior of the Solar Flux Density



Samples of the NOAA Web Site Solar Data

(18 Feb and 11Jan)

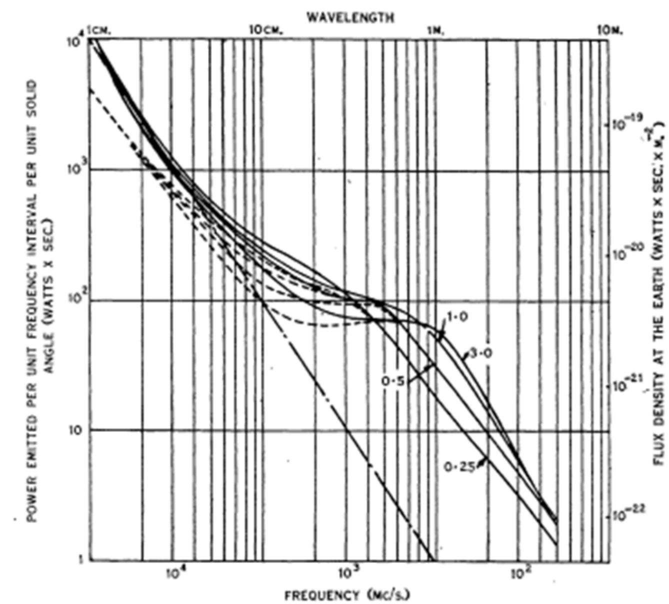
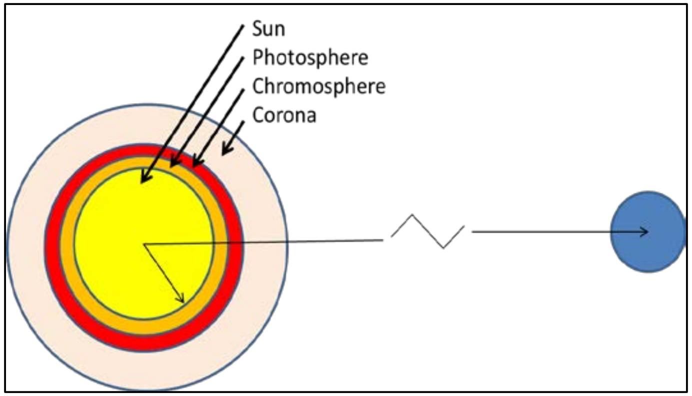


Fig. 7.—The variation of the amount of solar radiation with frequency for different chromospheric and coronal temperatures. The numbers on the curves refer to the coronal temperature in 10^6 °K. Two values of chromospheric temperature are used: $T_{ch} = 3 \times 10^4$ °K. (continuous lines); $T_{ch} = 10^4$ °K. (broken lines). Black-body radiation at any one temperature would appear in this figure as a straight line parallel to the line shown thus ——·——·——

Smerd 1950 Article on Solar Flux Density Predictions

RF Radiation from the Solar Atmosphere

- Solar astronomers tell us the sun is a G class star with a mean radius of about 695,000 km. The surface temperature is approximately 5900K by best fit thermal radiation theory curve with an exo-atmospheric total irradiance of 1353 Wm^{-2} at the earth, a distance 149.68e+6 km away. For optical radiation, including visible wavelengths, the solar surface is modeled as an ideal thermal radiator at 5900K. The model works quite well for optical astronomy.



- A technical article has been found that sheds a lot of light on solar RF properties. The Smerd Australian paper, circa 1950, is an "oldie but goodie."
  - Smerd indicates that both the chromosphere and the corona contribute appreciably to the emergent RF solar flux density as seen at the earth. For example, his estimate for 3000 MHz is roughly 50% from each of those two solar atmosphere layers.
  - Smerd also says the estimated electron temperature for the corona, at the time and still, was about 10^6 K. The chromosphere was thought to be around 10^4 K, which is also reasonably consistent with modern values.

- No wonder the solar surface model does not work well.

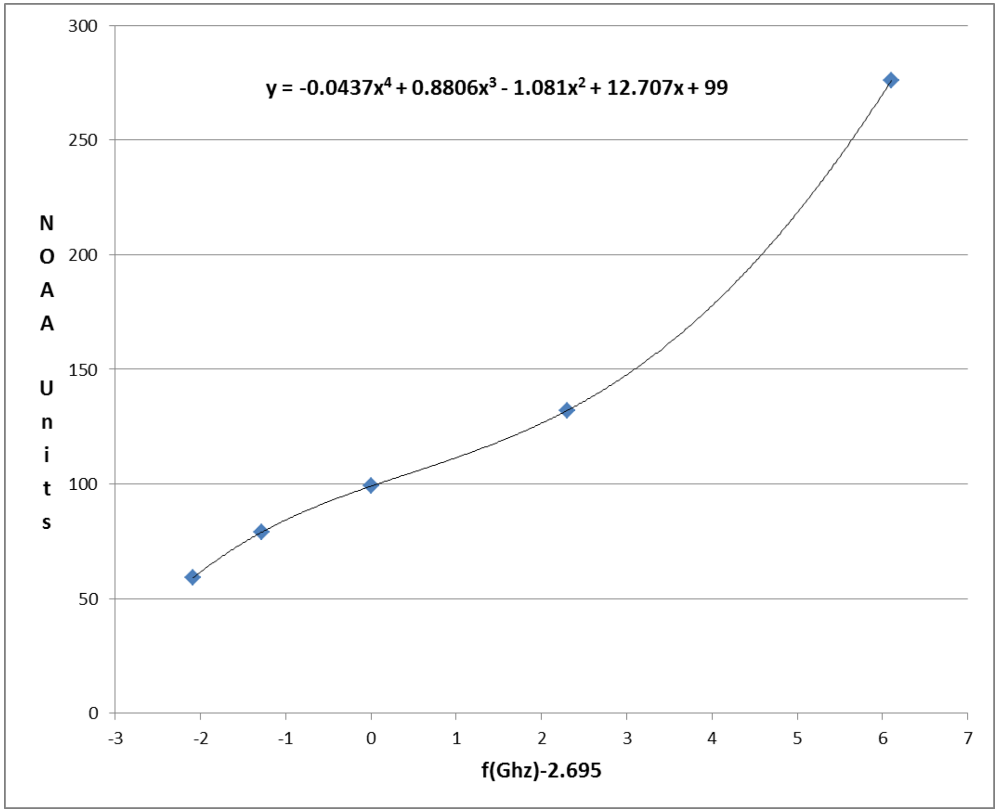
A Polynomial Function Fit to the NOAA Data

- Many RF astronomy groups, as well as airborne mobile telemetry (AMT) folks, require solar calibration. For example, the Search for Extraterrestrial Intelligence (SETI) Web Site recommends the following.
  - "If you are not operating very close to one of the eight NOAA frequencies, but instead between two given frequencies, then you will need to interpolate between flux densities at the lower and higher frequencies. The best interpolation scheme is to graph the NOAA flux data at several frequencies and use a curve fitting routine to determine the flux at your operating frequency."

$$poly = a + b\Delta f + c\Delta f^2 + d\Delta f^3 + e\Delta f^4$$

  - In this expression, poly is the function; a, b, c, d, and e, are the coefficients whose values are to be adjusted to obtain the best fit. The quantity Δf is the frequency variable measured from an arbitrary center frequency which may be chosen for convenience. For example, we will chose Δf = f – 2695 MHz. We will also find it convenient to express the frequency in GHz. The convenience of these steps makes the final adjusted coefficients have the highest possible number of significant figures in the EXCEL text results.
- The Microsoft EXCEL fitting procedure is called Trendline Analysis. The detailed steps of the procedure are given in Appendix C of the report.

Fit Results of the Forth Order Polynomial to NOAA Data of 18 Feb



Comparison of the Various Interpolation Methods

- We will compare methods by using the published NOAA flux densities only.
- We use every other frequency point as the two high/low frequencies and interpolate to get the third, or test point solar flux, at the intermediate NOAA frequency.
- We can then compare the interpolated result, at the intermediate frequency, with the given NOAA truth value.
- Thus the real solar data set provided from NOAA will suffice for our interpolation comparison methodology with no measurement results of our own required.

Polynomial and NOAA 18 February Data

Methodology

- The results of the plotted fit procedure in are shown in below.
- In this case the errors are the actual differences between the 18 February NOAA data values and the quartic polynomial fitted result values from the EXCEL procedure.
- The root mean square (RMS) of the errors is shown to be 0.021%.

Forth Order Polynomial EXCEL Trendline					
f (MHz)	S(f) (NOAA Units)	**Fit f (GHz-2695)**	To Be Fit S(f) (NOAA Units)	Fitted S(f) (NOAA Units)	Difference (NOAA Units)
245	20				
410	39				
610	59	**-2.085**	59	59.0	-1.0E-03
1415	79	**-1.28**	79	79.0	-1.3E-04
2695	99	**0**	99	99.0	0.0E+00
4995	132	**2.3**	132	132.0	-1.0E-03
8800	276	**6.105**	276	276.0	-4.7E-02
15400	581				
				RMS (%) =	**0.021**

RCC Formula Results (11 Jan and 18Feb)

Methodology

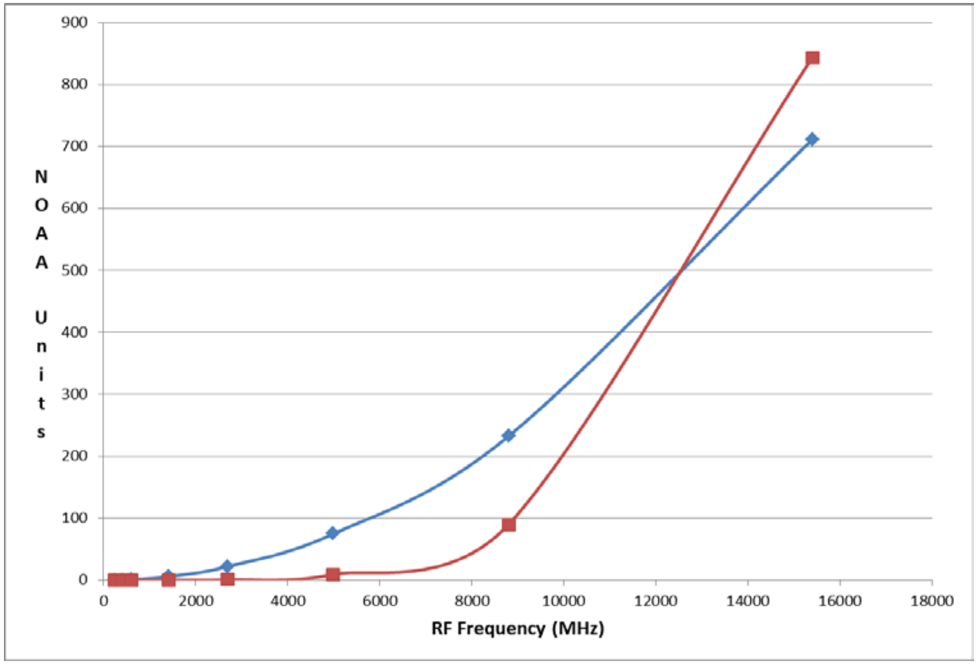
- The two tables show results when the RCC interpolation is applied to NOAA data of 11 January and 18 February.
- Every other S point pair is used to estimate the S at the intermediate frequency.
- The RMS results, 7.15% and 11.6%, are not impressive.
- The C-Band (4495 MHz) results are especially poor.

RCC Interpolation Formula Results (11 Jan)					
Point #	Frequency (MHz)	11 Jan Flux (NOAA Units)	Gamma	RCC S Result (NOAA Units)	Difference (NOAA Units)
1	245	36			
2	410	51	0.436	54.89	3.89
3	610	76	0.679	68.16	-7.84
4	1415	126	0.434	116.27	-9.73
5	2695	161	0.489	163.17	2.17
6	4995	209	0.479	233.33	24.33
7	8800	328	0.497	354.93	26.93
8	15400	599			
				RMS (%) =	**7.15**

RCC Interpolation Formula Results (18 Feb)					
Point #	Frequency (MHz)	18 Feb Flux (NOAA Units)	Gamma	RCC S Result (NOAA Units)	Difference (NOAA Units)
1	245	20			
2	410	39	0.436	36.83	-2.17
3	610	59	0.679	48.91	-10.09
4	1415	79	0.434	79.10	0.10
5	2695	99	0.489	102.69	3.69
6	4995	132	0.479	168.97	36.97
7	8800	276	0.497	278.16	2.16
8	15400	581			
				RMS (%) =	**11.63**

Power Law Simulation Plots

- One might ask how good the RCC interpolation formula results would be if the S_{RF} did obey a one term power law frequency dependence.
- The plot shows two simulation results that obey a quadratic power law (blue) and a forth power law (red).
- We use the interpolation formula for every other point as before to predict the point between.



Interpolation Results for 2nd and 4th Power Law Simulations

Methodology

- The results of the RCC interpolation formula applied to the two power law functions are shown in the tables.
- The results are essentially perfect.
- The derivation of the interpolation formula shows explicitly it is intended only for power law functions such as we have artificially generated in this section.
- Unfortunately the observed solar flux density does not follow such a power law.

RCC Interpolation Formula Results (f^2 Simulation)

Point #	Frequency (MHz)	f^2 Simulation (NOAA Units)	Gamma	RCC S Result (NOAA Units)	Difference (NOAA Units)
1	245	0.180			
2	410	0.504	0.436	0.504	0.00E+00
3	610	1.116	0.679	1.116	0.00E+00
4	1415	6.007	0.434	6.007	0.00E+00
5	2695	21.789	0.489	21.789	0.00E+00
6	4995	74.850	0.479	74.850	0.00E+00
7	8800	232.320	0.497	232.320	0.00E+00
8	15400	711.480			
				RMS (%) =	**0.00**

RCC Interpolation Formula Results (f^4 Simulation)

Point #	Frequency (MHz)	f^4 Simulation (NOAA Units)	Gamma	RCC S Result (NOAA Units)	Difference (NOAA Units)
1	245	5.40E-05			
2	410	4.24E-04	0.436	4.24E-04	0.00
3	610	2.08E-03	0.679	2.08E-03	0.00
4	1415	6.01E-02	0.434	6.01E-02	0.00
5	2695	7.91E-01	0.489	7.91E-01	0.00
6	4995	9.34E+00	0.479	9.34E+00	0.00
7	8800	9.00E+01	0.497	9.00E+01	0.00
8	15400	8.44E+02			
				RMS (%) =	**0.00**

NOAA Data and Simple Linear Interpolation

Methodology

- **For comparison, a linear interpolation method can be applied in a similar way.**
- **This method should indicate what a very simple and crude method is capable of.**
- **The formula used is**

$$S(f_2) = S(f_1) + \frac{S(f_3) - S(f_1)}{f_3 - f_1} \bullet (f_2 - f_1)$$

Linear Interpolation Formula Results (18 Feb)				
Point #	Frequency (MHz)	18 Feb Flux (NOAA Units)	S Result (NOAA Units)	Difference (NOAA Units)
1	245	20		
2	410	39	37.63	-1.37
3	610	59	46.96	-12.04
4	1415	79	74.44	-4.56
5	2695	99	97.95	-1.05
6	4995	132	165.68	33.68
7	8800	276	296.19	20.19
8	15400	581		
			RMS (%) =	**15.00**

- **The resulting errors are poor, but are only a factor of two worse than the RCC formula results.**

Discussion and Recommendations

- Suggestions for operational Implementation at Ranges
  - As done in the past, operator use the daily published NOAA flux density values to obtain the needed flux density at the test frequency.
  - However, now operator does not use the traditional RCC 118 interpolation formula to arrive at a flux value for his particular test frequency.
  - Instead, the operator would use the NOAA data values of that day with EXCEL Trendline Analysis to calculate the flux density at the test frequency.
  - Then, as usual, the calculated flux density value would then be plugged into the G/T formula to complete the calibration.
  - The above steps could be accomplished on an essentially manual basis.
  - A better approach might be an EXCEL computer utility, completely automated within EXCEL, so the operator would only need to enter the test frequency.
  - Appendix C in the report outlines the MS EXCEL steps that could accomplish the manual fitting and also provides steps that would help automate the process.
- Recommendations
  - Develop an automated, operator-friendly tool to fit a polynomial to NOAA data to get the solar flux density at the measurement frequency needed for the G/T calibration.

Derivation of RCC Interpolation Formula

Let us begin with the solar flux density formula, derived in Appendix A of the report, for solar surface model radiation, assumed in this case to be an ideal thermal source with a frequency squared power law dependence.

$$S_f = S_{RF} = \frac{\pi r^2}{R^2}\frac{2k_B T_{sun}}{c^2} f^2$$

$$\frac{S_f}{S_S} = \left[\frac{f}{f_S}\right]^2 \qquad \frac{S_L}{S_S} = \left[\frac{f_L}{f_S}\right]^2$$

$$\log\left[\frac{S_f}{S_S}\right] = \log\left[\frac{f}{f_S}\right]^2 = 2\log\left[\frac{f}{f_S}\right] \qquad \log\left[\frac{S_L}{S_S}\right] = \log\left[\frac{f_L}{f_S}\right]^2 = 2\log\left[\frac{f_L}{f_S}\right]$$

Now if we further divide the left equation by the right, we even cancel the factor 2.

$$\frac{\log\left[\frac{S_f}{S_S}\right]}{\log\left[\frac{S_L}{S_S}\right]} = \frac{2\log\left[\frac{f}{f_S}\right]}{2\log\left[\frac{f_L}{f_S}\right]} = \frac{\log\left[\frac{f}{f_S}\right]}{\log\left[\frac{f_L}{f_S}\right]} = \Gamma$$

[*Note: if the power 2 had been any other numerical power, such as 4, the same cancellation would occur. Thus the derivation works for all power laws, but only for powers of a single term, not more complex forms.*]

$$\log\left[\frac{S_f}{S_S}\right] = \Gamma\log\left[\frac{S_L}{S_S}\right] = \log\left[\frac{S_L}{S_S}\right]^\Gamma$$

Since $10^{\log X} = X$, we find,

$$10^{\log\left[\frac{S_f}{S_S}\right]} = 10^{\log\left[\frac{S_L}{S_S}\right]^\Gamma} \qquad \left[\frac{S_f}{S_S}\right] = \left[\frac{S_L}{S_S}\right]^\Gamma \qquad \boxed{S_f = \left[\frac{S_L}{S_S}\right]^\Gamma \bullet S_S}$$

Executive Summary

- TSIS implementation found issues with the solar G/T calibrations.
  - RCC-118-02, vol-2,App C - Solar Calibration.
  - An analysis step requires the use of an accepted interpolation formula.
- The formula assumes the frequency dependence of the solar flux density obeys a power law form, which is an untrue assumption.
- This study provides a comprehensive analysis of the interpolation formula and alternative methods.
- Real measurement data from NOAA web site reports is used in the study.
- Recommend fitting a simple polynomial function to the NOAA data to provide the needed interpolation.
- An error analysis is presented which demonstrates the merits of the various methods and supports the recommendation.