Capitalizing on all Test Data: Statistical Methods for Doing More without More

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Bottom Line Up Front

- **The purpose of this presentation is to illustrate proof of concept**
- **Support integrated testing**
  - How do we leverage all data in quantitative statistical analyses?
- **Results:**
  - Tighter confidence intervals
  - Better estimates
- **Future Directions**
  - Stryker case study shows value-added
  - How do we use this in future analyses?
  - How do we use this in scoping future test plans?

IDA

Outline

- **Motivation for Using All Information**
- **Statistical Methods for Combining Information**
- **The Stryker Family of Vehicles Case Study**
  - Methods
  - Exponential versus Weibull Distribution
  - Frequentist versus Bayesian Methodologies
  - Results
- **Extensions to effectiveness testing**
- **Conclusions**

Motivation For Using All Information

- **What is the Current Practice?**
  - DOT&E in most cases uses only operational test data for evaluation
    - » Benefit: ensures data is representative of operational test conditions
    - » Drawback: discards information from previous testing that provides information on system reliability
- **Why use all test data?**
  - Testing is expensive
  - Lose valuable information by not using all information
- **National Research Council Studies**
  - *Statistics, Testing and Defense Acquisition*, **1998**
    - » Emphasizes that all relevant information be examined for possible use in both the design and evaluation of operational tests …
    - » State-of-the-art statistical methods for combining information should be used, when appropriate, to make tests and their associated evaluations as cost-efficient as possible
  - *Improved Operational Testing and Evaluation*, **2006**
    - » Focuses specifically on methods of combining information for the Stryker family of vehicles

IDA Statistical Methods for Combining Information

- **Two primary avenues:**
  - Combine summary statistics from multiple analyses
  - Combine data and account for multiple events in the modeling
- **Combine information through summary statistics**
  - Averaging P-values: Fisher (1948), Pearson
  - Averaging test-statistics: Mosteller and Bush (1954), Stouffer (1949)
- **Combine information in the model**
  - Meta-Analysis/Frequentist approach
    - » Standard Analysis – combine all data pretend it comes from the same test
    - » Blocking by test phase – focus on between test variation
    - » Fixed Effect Meta-Analysis – allow for a shift in parameter values between tests but can only model within test variation
    - » Random Effect Meta-Analysis – general version that allows for both within test variation and between test variation to be modeled.
  - Bayesian hierarchical models

Model-based approaches allow for maximum flexibility!

Keys to Success

- **Eliminate or account for as many sources of variation as possible**
  - Common response variable across test phases:
    - » Reliability data
      - Consistent data collection and scoring
      - Detailed data records including:
        - Miles between each abort (not just total miles and total aborts)
        - Sub-system records for each abort
- **Leverage all common information**
  - Family of Vehicles: allows us to pool information by leveraging relationships between vehicles
- **Think hard about the model**
- **Understand the pedigree of your data**

The Stryker Family of Vehicles: A Case Study

Infantry Carrier Vehicle

Mortar Carrier Vehicle

Engineer Squad Vehicle

Stryker System Description

- **The Stryker family of vehicles includes 10 separate systems**

- **Two Basic Vehicle Variants**
  1. Infantry Carrier Vehicle (ICV) - the infantry/mission-vehicle type
     - Base vehicle for eight separate configurations
       - Infantry Carrier Vehicle (ICV)
       - Mortar Carrier Vehicle (MCV)
       - Antitank Guided Missile Vehicle (ATGMV)
       - Reconnaissance Vehicle (RV)
       - Fire Support Vehicle (FSV)
       - Engineer Squad Vehicle (ESV)
       - Commander's Vehicle (CV)
       - Medical Evacuation Vehicle (MEV)
       - NBC Reconnaissance Vehicle (NBCRV)*

     Considered in this analysis

  2. Mobile Gun System (MGS)* – direct fire platform and performs the maneuver fire support role

**NBCRV and MGS were not included because they were on a different acquisition timeline*

Stryker Mission Essential Functions

- **There are four essential functions**
  - Move
  - Shoot
  - Command and Control, Communications, Computers, Intelligence, Surveillance and Reconnaissance (C4ISR)
  - Survive
- **A failure is an event in which an item or part of an item does not perform as specified**
- **The Army failure definition scoring criteria (FDSC) categorizes the severity of failures**
  - **System Abort**
    - » The vehicle is unable to complete the mission
  - Essential Function Failure
  - Non-essential Function Failure
- **Reliability requirement:**
  - Mean miles between system aborts = 1,000 miles

Developmental and Operational Testing

Developmental Testing

- **Controlled Conditions**
- **Experienced Technicians operating the vehicles.**
  - They have done this for years and they know the courses really well
- **Courses**
  - Use courses that are designed to replicate the primary roads, secondary roads, and trail like conditions

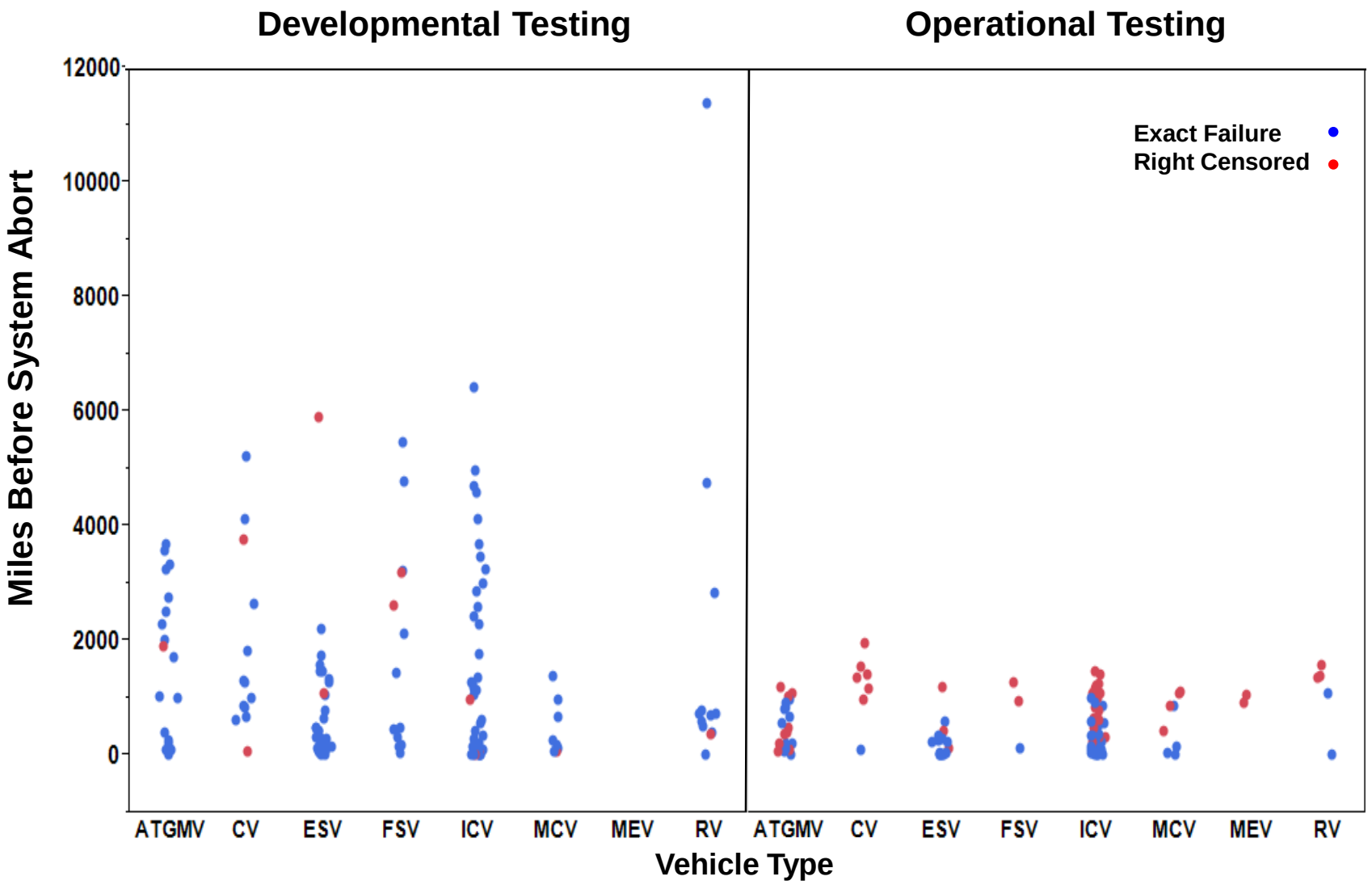
DT And OT Are Different!
- Operators
- Environments
- Test Durations

Operational Testing

- **Operational Conditions**
- **An army unit comes in to do this testing**

- **Courses**
  - OT data set comes from testing that was done at Fort Knox
  - Most of the testing was done using secondary road type conditions
- **Limited amount of Time**
  - Due to operator availability and range availability
  - Operational testing may be too short to discover many reliability deficiencies

The Stryker 2003 Data Set



A Traditional Analysis - Using OT Data Only

- **The table below is similar to that which was included in the report written for DOT&E when considering this data set.**

- **These results serve as the reference when comparing the new methods that look at combining information across the developmental and operational test phases.**

Stryker Reliability by Variant using Operational Test Data

Vehicle Variant	Total Miles Driven	System Aborts	MMBSA	MMBSA 95% LCL	MMBSA 95% UCL
ATGMV	10334	12	861	492.9971	1666.62
CV	8494	1	8494	1524.505	335495.1
ESV	3771	13	290	169.6326	544.7885
FSV	2306	1	2306	413.8815	91082.13
ICV	29982	35	857	615.9437	1229.84
MCV	4521	4	1130	441.4354	4148.219
MEV	1967	0	-	656.6007	-
RV	5374	2	2687	743.8384	22187.42
Total	**66749**	**68**	**982**	774.2946	1264.074

$$\text{Mean Miles Before a System Abort } (\mathbf{MMBSA}) = \frac{\text{Total Miles Driven}}{\text{System Aborts}}$$

Failure-Time Regression Models

We began by using the exponential distribution to model the miles before a system abort

$$t_{ijk} \sim exponential(\lambda_{ij})$$

$i = 1,2$ (test phase)
$j = 1,2,\ldots,7$ (vehicle variant)
$k = 1,2,\ldots,n_{ij}$ (miles)

We can express rate parameter, λ, as a function of explanatory variables to find estimates for the MMBSA

Model 1:

Average over vehi… …es vehicle type does not matter)

$$\lambda_{i.} = \gamma_0 + \gamma_1 \text{Test Phase}$$

Naïve : we know variant and test phase impact reliability

Model 2:

Average over test phase (assumes test phase does not matter)

Yes, we combine information – but we completely ignore the test phase!

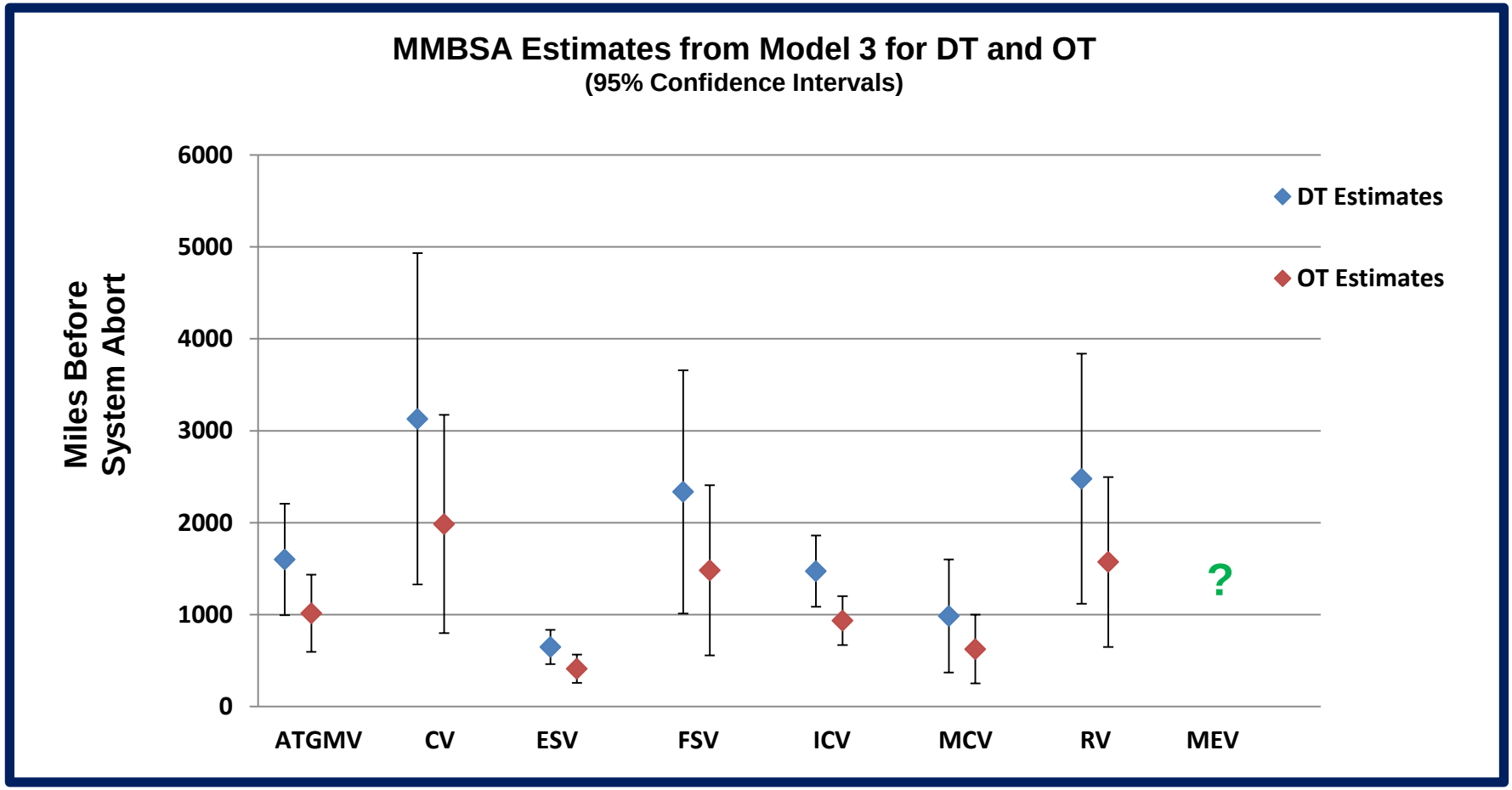
$$\lambda_{.j} = \gamma_0 + \gamma_1 \text{ATGMV} + \ldots + \gamma_6 \text{MCV}$$

Model 3:

Look at differences based on Test Phase & Vehicle Type

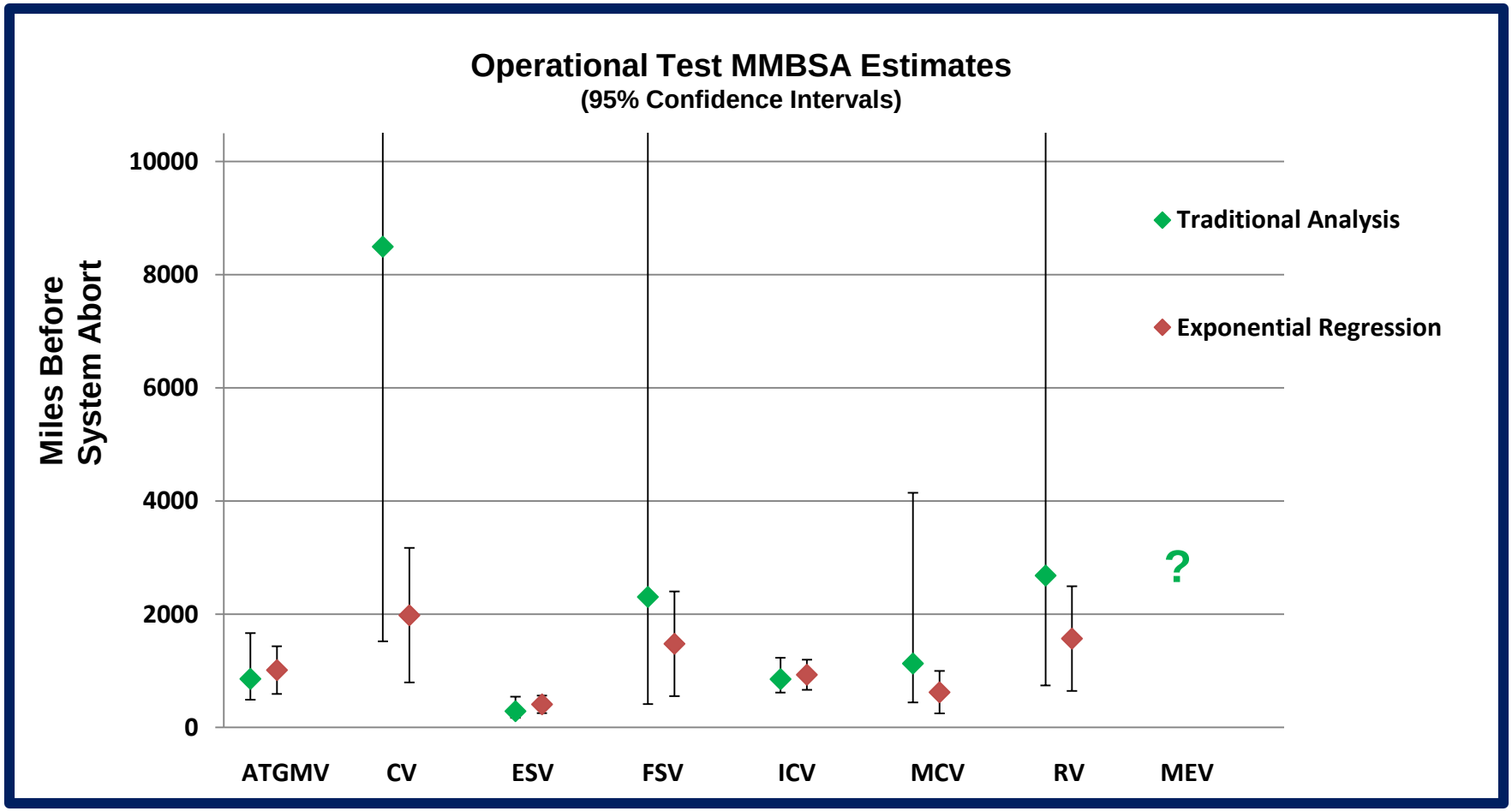
$$\lambda_{ij} = \gamma_0 + \gamma_1 \text{Test Phase} + \gamma_2 \text{ATGMV} + \cdots + \gamma_7 \text{MCV}$$

Exponential Regression Results



This model estimates a 37% reduction in the MMBSA moving from DT To OT

Comparing Confidence Intervals



Tighter confidence intervals & better estimates for MMBSA

Incorporating More Information Bayesian Approach

- **Informative Priors**
  - Based on subject matter expertise
    - » Data is already included in model
- **Hierarchical Models**
  - Assumes the parameters are related, the data tells us how closely related
  - Hierarchical models for the Stryker case study allow us to estimate MEV reliability based on other data

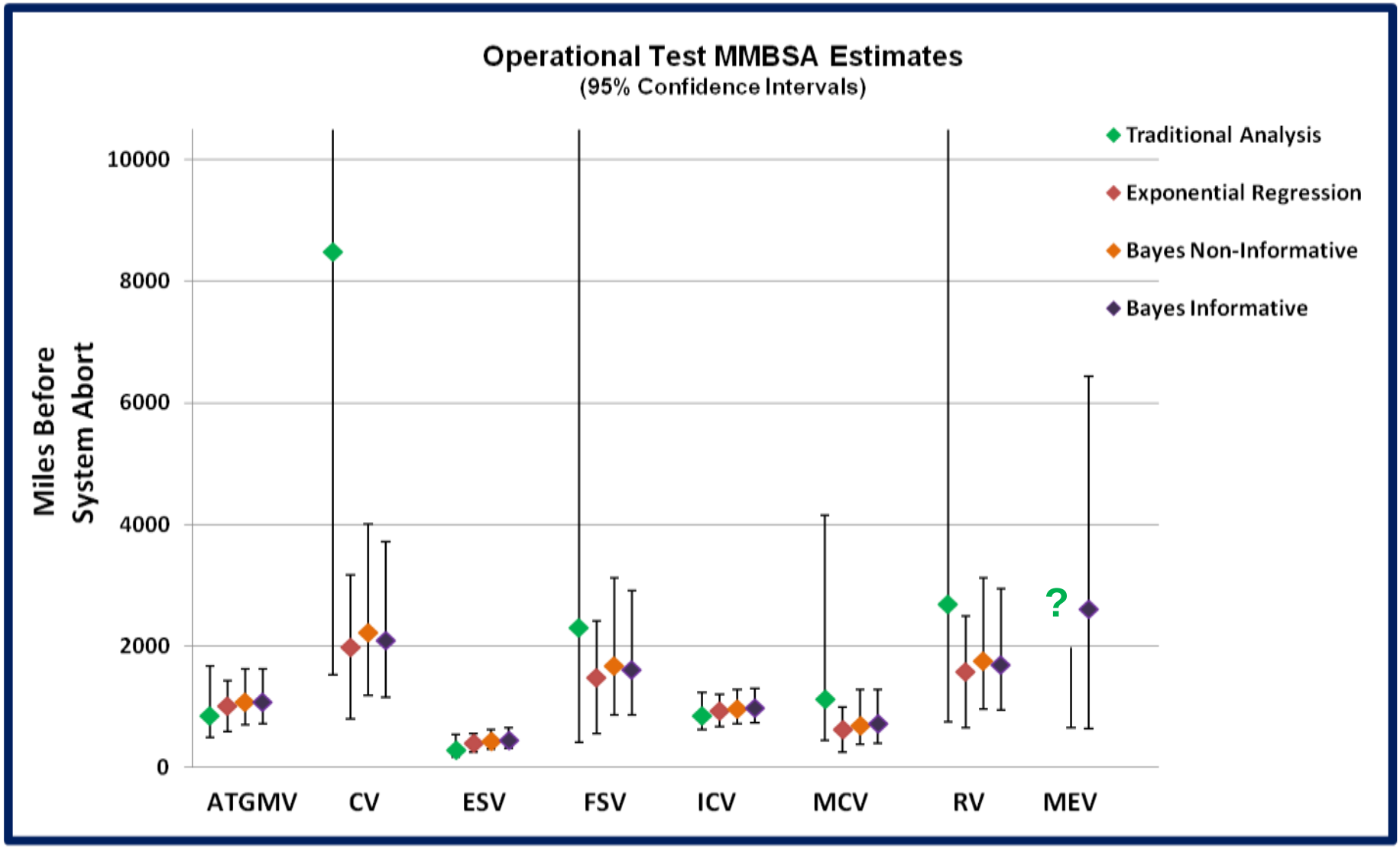
A Model That Allows Us To Estimate MEV Reliability

$$t_{DT} \sim exp(\lambda_i) \quad t_{OT} \sim exp(\lambda_i/\eta)$$

$i = 1,2,\ldots,8$ (vehicle variants including MEV)

$$\lambda_i \sim gamma(a,b)$$
$$\eta \sim beta(1,1)$$
$$a \sim gamma(.001,.001)$$
$$b \sim gamma(.001,.001)$$

Incorporating More Information



Statistical Challenges

- **Likelihood based inferences**
  - Cannot always be done in standard statistical software
  - Multivariate delta method
- **Censored data**
- **Might need to write your own code.**
  - Software packages don't always provide enough flexibility
- **No data set is ever perfect**
  - Missing data
    - » Multiple imputation
    - » Bayesian imputation

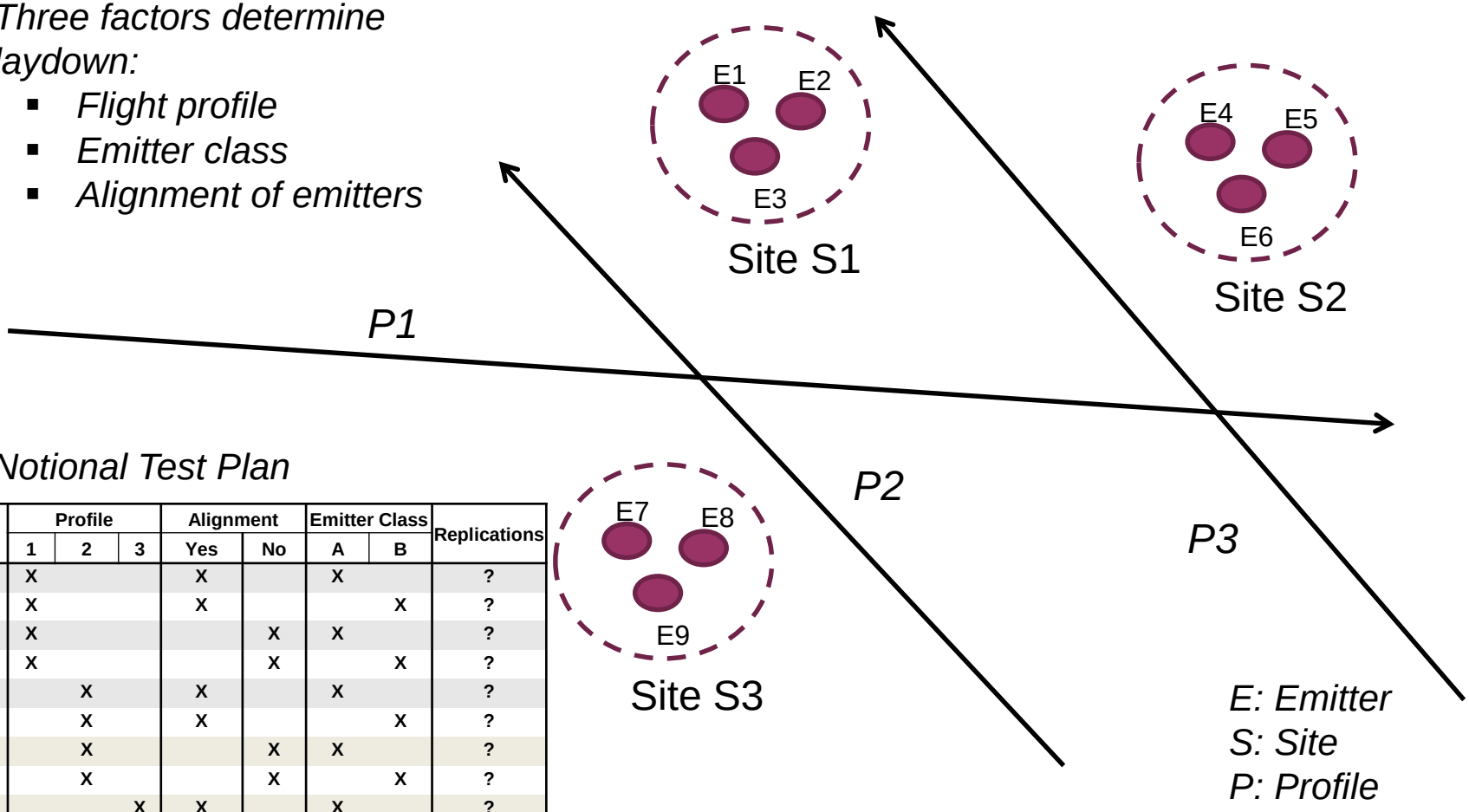
Extensions to Effectiveness Testing

- **Common Challenges:**
  - Common response variables across multiple phases of testing
  - Accounting for test conditions
- **Example: Next Generation Jammer (NGJ)**
- **Objectives:**
  - Combine information across software in the loop, hardware in the loop, modeling and simulation, and open air testing
  - Challenge: high density open air testing is impossible

Response Variable Matrix				
Threat Scenario	Venue			
	SIL	HIL	Open Air	M&S
One-on-one	JRT	JRT	N/A	N/A
Low Density	JRT	JRT	JRT, ETT	JRT, ETT
High Density	JRT	JRT	N/A	JRT, ETT
Key Response Variables:	JRT- Jam Response Time, ETT-Engagement Track Time			

NGJ: Emitter/Flight Profile Laydown

- *Three factors determine laydown:*
  - *Flight profile*
  - *Emitter class*
  - *Alignment of emitters*



Notional Test Plan

Laydown	Profile			Alignment		Emitter Class		Replications
	1	2	3	Yes	No	A	B	
1	X			X		X		?
2	X			X			X	?
3	X				X	X		?
4	X				X		X	?
5		X		X		X		?
6		X		X			X	?
7		X			X	X		?
8		X			X		X	?
9			X	X		X		?
10			X	X			X	?
11			X		X	X		?
12			X		X		X	?

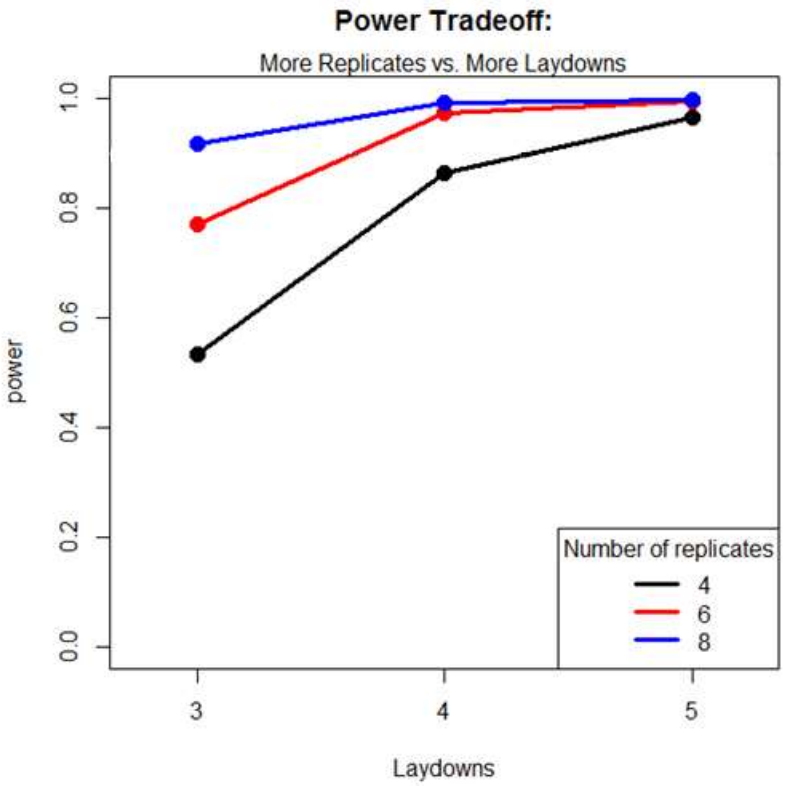
Test Design Approach

- **Goal is to design a test that supports combining information**
  - 12 Laydowns of interest
  - Combining information can reduce replication

ANOVA Model:

$$Y_{ijkl} = \mu + \alpha_i + \beta_j + C_k + e_{ijkl}$$

Overall mean: μ
α_i: JRT
β_j: Density
C_k: Venue
Laydown
e_{ijkl}: Random error



	Test Venue			
	SIL	HIL	OAR	M/S
1 on 1				
Low				
High				

Combining Information reduces replication from 6 to 3 replicates – Cutting the open air test in half!

Conclusions

- **We can use basic statistical models to incorporate information from multiple testing phases into OT evaluations**
- **The results are:**
  - Tighter confidence intervals
  - Better estimates
  - Benefits are greatest when only limited OT data is available
- **Model specification requires careful consideration**
  - If the model is wrong the results are not meaningful
- **Bayesian techniques provide:**
  - Ability to incorporate more information than is contained in the data
    - Subject matter expertise
    - Historical information not directly contained in data
  - Ease of inference
    - Missing data imputation
    - Censored data with complex likelihoods
- **Analysis requires more statistical knowledge than the traditional OT analyses**
  - Information gained is worth the effort

Caveats and Future Directions

- **Concerns**
  - Need both statistical and system engineering expertise to make this work
  - Model specification is key, the model must be appropriate for the data
  - Analyses are nontrivial compared to current standard analyses
- **Combining information applies to more than reliability**
  - Need common measures and conditions across test phases
- **Future Directions**
  - How do we use this in future analyses?
  - How do we use this in scoping future test plans?

Questions?

Backup Slides

Reliability Analysis

- **Reliability is an essential component of the assessment of operational suitability**
- **Examples of reliability data:**
  - » Miles driven until failure, hours of use until a failure, number of on-off cycles until a failure
- **Commonly used distributions in reliability:**

Exponential Distribution

- Historically used in DoD reliability assessment
- Simple model: only one parameter to estimate

$$f(t_i) = \frac{1}{\lambda} e^{-\left(\frac{t_i}{\lambda}\right)}$$

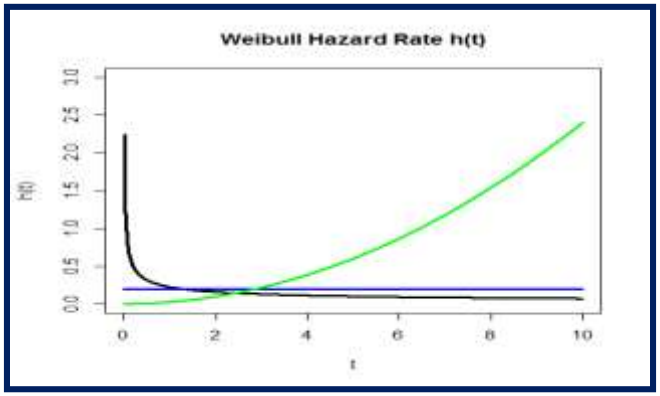
- Easy to interpret: under this parameterization, λ is the mean time between failures

Weibull Distribution

- Flexible distribution: two parameters

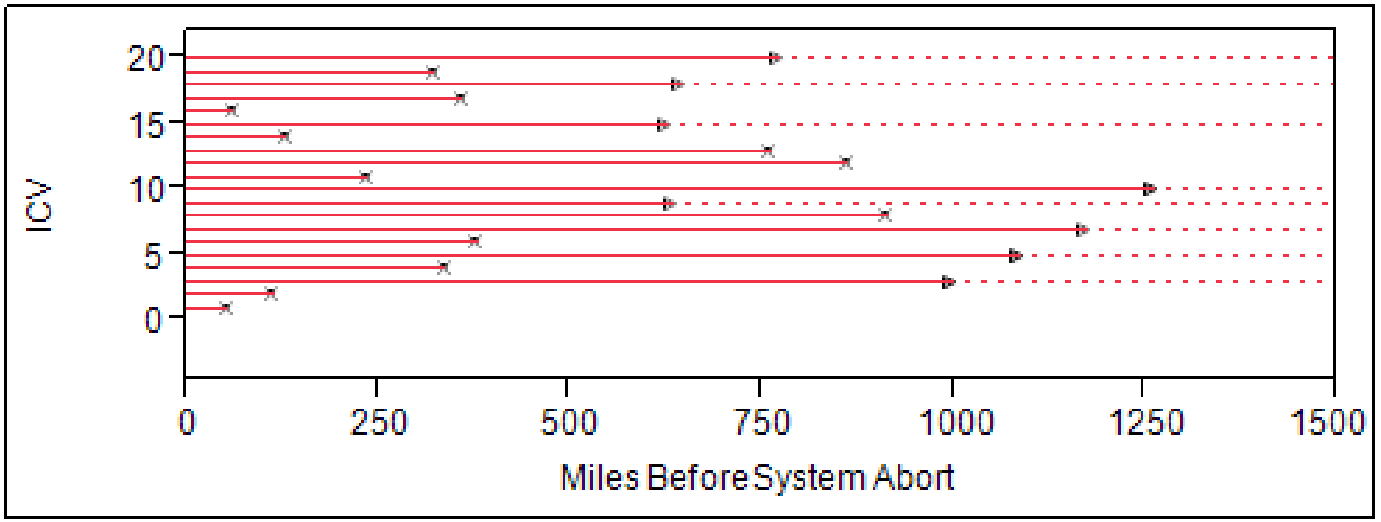
$$f(t_i) = \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_i}{\eta}\right)^{\beta}}$$

- Can describe multiple failure mechanisms



Unique Features of Reliability Data

- **The exact failure times are not always known.**
  - When this happens we say that the data is censored



- **Censoring is accounted for in the Likelihood**
- **No negative data values (failure times > 0)**
  - We model reliability data using distributions for positive random variables
  - The exponential and Weibull distribution are two common choices

Bayesian Analysis

- **Bayesian models still require a parametric statistical model**
  - Bayesian model is specified by:
    - » Parametric statistical model (just as before)
    - » Prior distribution
  - Bayes Theorem: posterior distribution is proportional to the likelihood (data) times the prior

- **Why might we want to consider this option?**
  - Incorporate more information through the use of a prior
    - » A degradation from DT to OT
    - » This allows for us to come up with an estimate for the Medical Evacuation Vehicle (0 observations in DT and 2 censored observations in OT) by using the information that we know about the other vehicles.
  - Ease of inference

We can incorporate more information!

Bayesian Models Considered

Bayesian Model 1

$$t_{DT} \sim exp(\lambda) \quad t_{OT} \sim exp(\lambda/\eta)$$

Using Non-Informative Priors:

$$\lambda \sim gamma(.001, .001)$$
$$\eta \sim beta(1,1)$$

Comparable to the Failure-time Regression Model 1

Bayesian Model 2

$$t_{DT} \sim exp(\lambda_i) \quad t_{OT} \sim exp(\lambda_i/\eta)$$

$i = 1,2, \ldots, 7$ (vehicle variants)

Using the Non-Informative Priors:

$$\lambda_i \sim gamma(.001, .001)$$
$$\eta \sim beta(1,1)$$

Comparable to the Failure-time Regression Model 3

Bottom Line Up Front

- **The purpose of this case study is to illustrate proof of concept**
  - Stryker OT dataset is robust
  - Common chassis, multiple variants
- **Support integrated testing**
  - How do we leverage all data in quantitative statistical analyses?
- **Results:**
  - Tighter confidence intervals
  - Better reliability estimates
  - Benefits are greatest for vehicles with only 0-2 reported failures in OT
- **Future Directions**
  - Stryker case study shows value-added
  - How do we use this in future analyses?
  - How do we use this in scoping future test plans?

More on Reliability…

- **Reliability is an essential component of the assessment of operational suitability of major defense systems**
- **We can think of reliability <u>as quality over time</u>**

  One comes to expect that a system, vehicle, machine, or device will perform its intended function under its appropriate operating conditions for some specified period of time.

- **We use data to help predict and assess various aspects of product reliability**
- **Some examples of reliability data include:**

  Miles driven until failure, hours of use until a failure, number of on-off cycles until a failure, …

Failures Are What We Care About

Model Selection Considerations

- **Ease of use**
  - Exponential regression available in JMP
  - Bayesian techniques require code writing
  - Explanation of results
- **Frequentist versus Bayesian**
  - Interpreting confidence intervals (credible intervals)
  - Zero failures – point estimates only exist in a Bayesian framework
  - Can we incorporate information from data directly?
    - » Bayesian models allow us to incorporate information only available as summary statistics
- **Informative versus Non-informative priors**
  - Is there reliable subject matter expert information to incorporate?

Weibull Model Specification

- **Weibull distribution has two parameters, β and η**

$$f(t_i) = \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_i}{\eta}\right)^{\beta}} \qquad F(t_i) = 1 - \exp[-\left(\frac{t_i}{\eta}\right)^{\beta}]$$

  - Both parameters could be impacted by test phase (DT/OT) and vehicle variant
  - Considered two models:
    - » Both β and η as a function of variant and test phase
    - » Only η as a function of variant and test phase
  - Test phase did not impact the model shape parameter, β

- **The Weibull Regression Model**

$$\mu_{ij} = \log(\eta_{ij}) = \gamma_0 + \gamma_1 Test\ Phase + \gamma_2 ATGMV + \cdots + \gamma_7 MCV$$

  - Estimating the model parameters: $\gamma_0, \gamma_1, \gamma_7, \beta$

Exponential Model Specification

- **Weibull distribution has two parameters, β and η**

$$f(t_i) = \frac{1}{\lambda} e^{-\left(\frac{t_i}{\lambda}\right)} \qquad F(t_i) = 1 - \exp[-\left(\frac{t_i}{\lambda}\right)]$$

- **The Exponential Regression Model**
  - Recall that we considered three models:
    - » The Most Appropriate Model

$$\lambda_{ij} = \gamma_0 + \gamma_1 \text{Test Phase} + \gamma_2 \text{ATGMV} + \cdots + \gamma_7 \text{MCV}$$

  - Estimating the model parameters: $\gamma_0, \gamma_1, \cdots, \gamma_7$

Failure-Time Regression Models: Censoring and MLE

- **We need to estimate the regression model parameters!**
  - We do this using Maximum Likelihood Estimation (MLE)
    - » The estimates for the model parameters are the values that maximize the likelihood function

- **Total Likelihood for right censored data**
  - Product of the likelihood contributions:

$$L(\boldsymbol{\Theta}|\mathrm{t}_1, \ldots, \mathrm{t}_\mathrm{n}) = C \prod_{i=1}^{n} [f(t_i)]^{\delta_i} [1 - F(t_i)]^{1-\delta_i},$$

Exact Failure

Right Censored Contribution

  - Where:

$$\delta_i = \begin{cases} 1 \text{ exact failure} \\ 0 \text{ right censored} \end{cases}$$

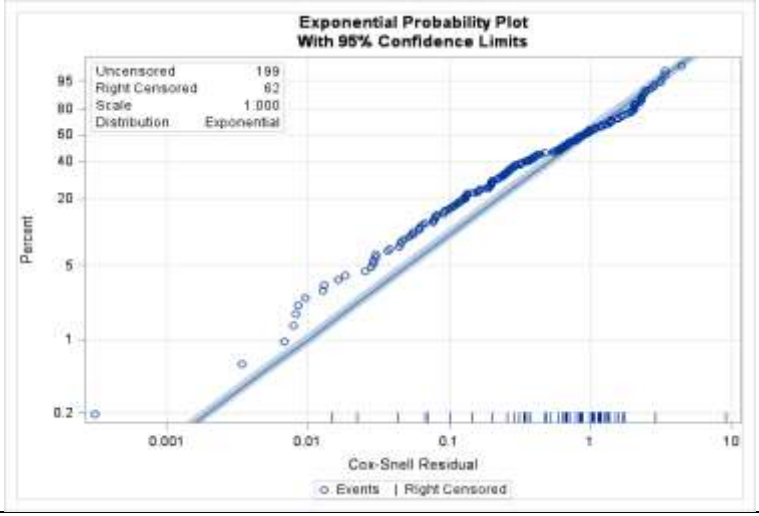
$\boldsymbol{\Theta}$ is a vector of model parameters

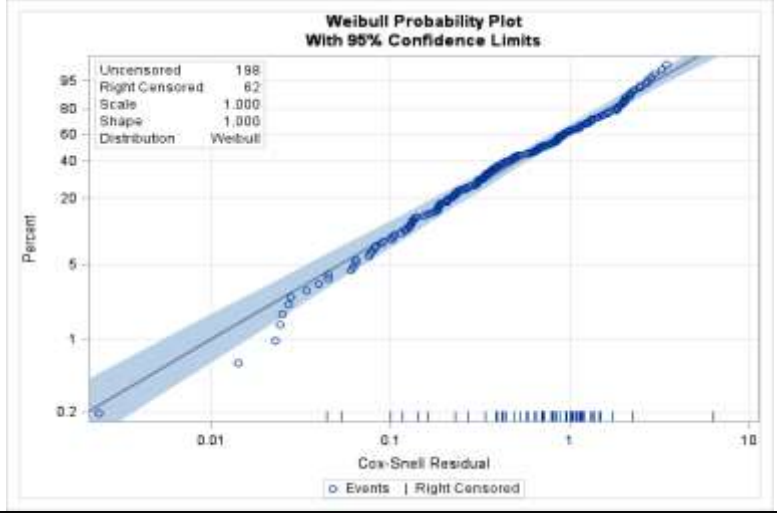
$f(t_i)$ is the pdf for the distribution under consideration

$F(t_i)$ is the cdf for the distribution under consideration

Assessing The Model Adequacy Of Failure Time Regression Models

- **Model Comparisons**
  - Weibull is the best distribution to use based on the model comparison AIC and BIC values.
- **A Whole Model Test**
  - Exponential Regression: p < .0001
  - Weibull Regression: p< .0001
- **Probability Plots of Residuals for Exponential and Weibull Regression**





MCMC Routine

- **The Steps below are outlined under the assumption that the data follows a Weibull distribution (easy to modify for exponential distribution)**

- **Calculate the Log-Posterior:**

$$= logL(\gamma_0, \dots \gamma_7, \gamma_8, \beta | \mathrm{t}_1, \dots, \mathrm{t}_\mathrm{n}) + \sum_{i=1}^{8} \pi(\gamma_i) + \pi(\beta) + \pi(\eta)$$

- **Algorithm**

Step 0:
Initialize starting value for $\gamma_1, \gamma_2, \gamma_3, \dots \gamma_7, \beta, \eta, t_{missing}$

Step 1:
Propose γ_1 -> accept or reject using log-posterior (using current values of other parameters).
Propose γ_2 -> accept or reject using log-posterior (updated γ_1 value and current values of other parameters).
" " … for other parameters $(\gamma_3, \dots \gamma_7, \beta, \eta)$

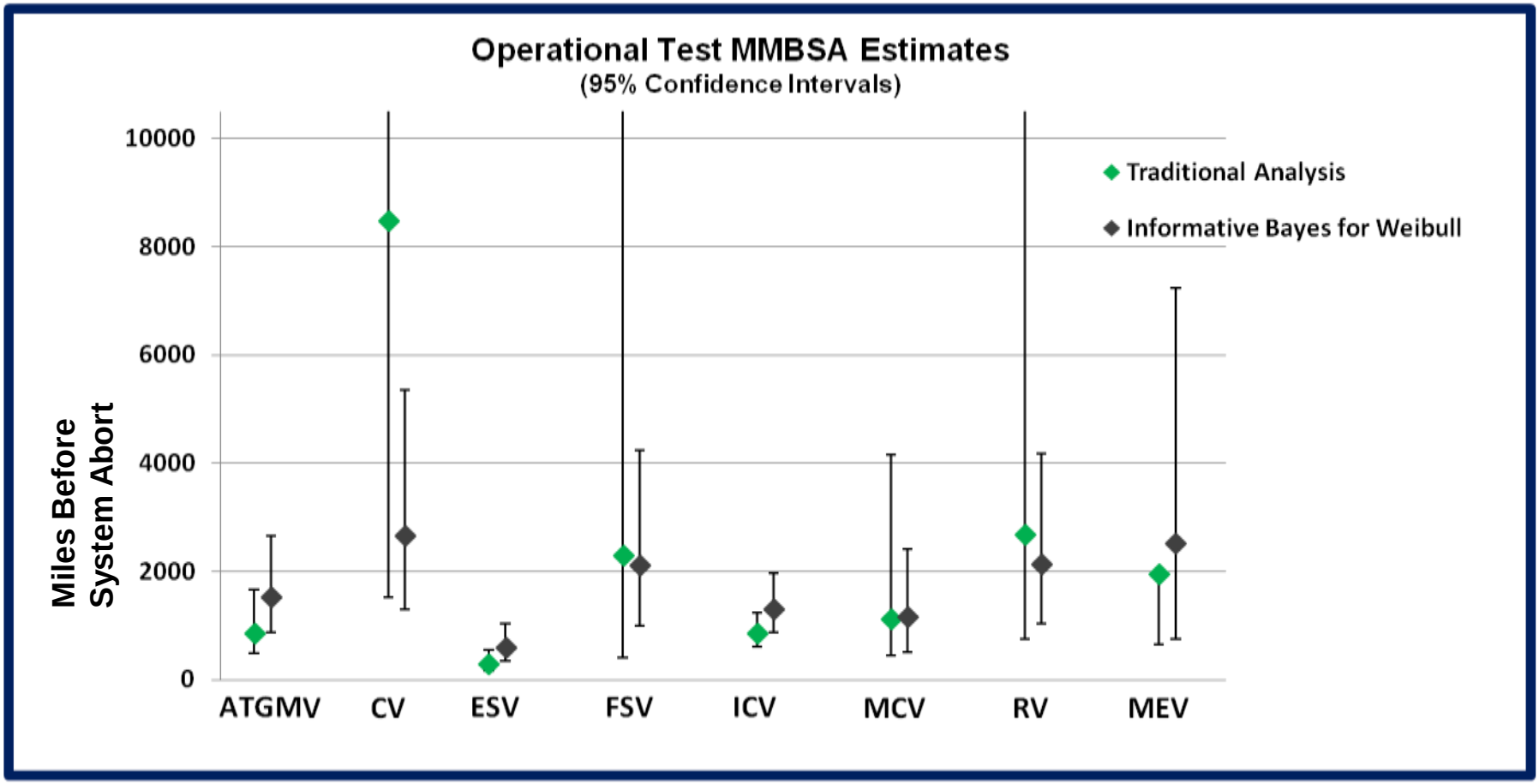
Step 2:
Update missing data and adjust the other failure times accordingly. In this step we can sample using the fact that:

$$t_{missing} \,|\, \gamma_{phase,variant}\,, \beta \sim Weibull\,(\gamma_{phase,variant}\,, \beta)$$

Step 3:
Repeat Steps 1 and 2 a total of N times.

Results: Incorporating More Information

We can use Bayesian methods for t ~ Weibull too!



Summarizing Confidence Intervals

Vehicle	Reduction in Intervals (compared to Traditional Analysis) Under the Assumption t ~ Exponential
ATGMV	0.25
CV	0.99
ESV	0.13
FSV	0.98
ICV	0.10
MCV	0.77
RV	0.91
MEV	
Column Average	0.59

A Traditional Analysis - Using DT Data Only

Stryker Reliability by Variant using Developmental Test Data					
Vehicle Variant	Total Miles Driven	System Aborts	MMBSA	MMBSA 95% LCL	MMBSA 95% UCL
ATGMV	30086	17	1770	1105	3038
CV	24160	11	2197	1228	4400
ESV	25095	35	717	516	1029
FSV	24385	11	2217	1239	4441
ICV	61623	39	1580	1156	2222
MCV	3702	7	529	257	1315
MEV	-	-	-	-	-
RV	23742	11	2158	1206	4324
Total	**192793**	**131**	1472	1240	1760

$$\text{Mean Miles Before a System Abort } (\mathbf{MMBSA}) = \frac{\text{Total Miles Driven}}{\text{System Aborts}}$$

Comparing Traditional Results For DT And OT To Exponential Regression Results